Singularity Analysis for 3-DoF (P-4R) Parallel Manipulators Based on Screw Theory

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***Abstract*—In this paper, structure singularities and actuated joints configuration singularities analysis presented for a new family of 3-DoF P-4R parallel manipulators, using the screw theory, reciprocal screw and linear geometric theory. Geometric conditions used as guildlines for the design of parallel mechanisms that can be avoid architecture singularities. The effectiveness method used to analyze the same special parallel manipulators. Based on these analysis theory, numerical examples with certainly displacement of singularity are provided.**

***Index Terms*—singularity, screw theory, reciprocal screw, parallel manipulator**

CLC number: TP24

I. INTRODUCTION

Parallel manipulators was first introduced in tire testing by Gough and were later on used by Stewart as flight simulators [1].In the later two decades, researches have drawn great attention to parallel manipulator, because of the attractive performance. They exhibit in many fields (ranging from robotics to parallel kinematics machine tool) by virtue of valuable features such as low inertia, high payload to manipulator weight ratio, considerable stiffness and high dynamic performance. However, many applications of a six-degree-of-freedom (DoF) mechanism are not required and lower-DoF manipulators may be used, especially the three-DoF manipulators. Fang [2] used the theory of reciprocal screws for the enumeration of feasible limb structures of a class of 3DoF rotational parallel manipulators, he concluded a table of feasible limbs that can be used for the construction of 3-DoF rotational parallel manipulators. Joshi [3] enumerated a class of 3-DoF, 4-legged parallel manipulators and presented a methodology for analyzing the kinematics of such manipulators. Kong [4] proposed a method for the type synthesis of 3-DoF translational parallel manipulators based on screw theory. He also proposed the validity condition of actuated joint of this mechanism. Miller [5] investigated the influence of motor axes orientation on the workspace volume of 3-DoF manipulators. There also are many of literatures mentioned about 3-DoF parallel manipulators [6~8]. In [8], a new architecture for a 3-DoF translational parallel manipulator had been presented. The moving platform is connected to the base by three limbs, each limb composed by four revolute joints, parallel two by two, and a prismatic joint which actuated in limb and be mounted in certainly displacement.

In this paper, we analyze the singularity of this mechanism with actuated singular and displacement (structure) singularity. In section 1, we review the screw and reciprocal theory. A displacement and actuated singularities be presented in section 2. Finally, numerical examples with certainly displacement and actuated singularities are provided in section 3, section 4 is the conclusion of this paper.

1 screw theory and reciprocal screw

This section presents a short summary of the reciprocal screw theory and a null-space approach for the determination of reciprocal screws. A unit screw is defined as a 6×1 column matrix [9].

$$\hat{\$}_i = \begin{bmatrix} s_i \\ r_i \times s_i + \lambda s_i \end{bmatrix} \tag{1}$$

where s_i is a unit vector pointing in the direction of the screw axis, r_i is the position vector of any point on the joint axis with respect to a reference frame, and λ is the pitch of the screw. $r_i \times s_i$ defines the moment of the screw axis about the screw. For a revolute joint, $\lambda = 0$, and for a prismatic joint, $\lambda = \infty$. The subscript i is used to indicate the screw associated with i th joint.

Applying the reciprocal condition to each unit joint screw, we obtain

$$\hat{\$}_r^{\ T} \$_i = 0 \tag{2}$$

where $i = 1,2....,6-n$ and $r = 1,2,..n$, the transpose of a screw is defined as follow:

$$\hat{\$}_r^{\ T} = \begin{bmatrix} s_{r4} & s_{r5} & s_{r6} & s_{r1} & s_{r2} & s_{r3} \end{bmatrix}$$

So that the Eq.(2)can be denoted as

$$\hat{\$}_r^T \$_i = s_{r4}s_1 + s_{r5}s_2 + s_{r6}s_3 + s_{r1}s_4 + s_{r2}s_5 + s_{r3}s_6 \quad (3)$$

where $s_{ri}(i=1,2,....6)$ is the ith component of the rth unit screw and $\$_i(i=1\sim 6)$ is the reciprocal screw of the $\hat{\$}_r$.

II. SINGULARITY ANALYSIS

Evaluation of singularitics plays an important roles in parallel manipulators including design, trajectory planning, and control. A parallel manipulators is said to be in a singular configuration when the manipulator jacobian matrix loses rank. In this section, we analyze two singularities: displacement singularity and actuated singularity of P-4R translation parallel manipulator.

A Displacement singularity

The mechanism consists of a moving platform connected to a fixed base by three symmetrical similar limbs. Fig. 1 shows the structure of this mechanism. Each limb containing four revolute passive joints and one prismatic actuated joint which can be mounted anywhere along the kinematic chain. In this article, we discussed this situation that the actuated joint be mounted on the base in order for conveniency actuated. Fig. 2 shows the kinematic model of the $i-th$ limb, we name the four revolute passive joints R_{1i} , R_{2i} , R_{3i} and R_{4i} respectively and a prismatic actuated joint P_i. The axes of R_{i1} and R_{i2} and axes of R_{i3} and R_{i4} are mutually parallel respective. Assumed that the direction of axes R_{i1} and R_{i2} denoted as $s_{i1}=(m_{i1},n_{i1},p_{i1})$, simultaneity, the direction of axes R_{i3} and R_{i4} denoted $s_{i2}=(m_{i2},n_{i2},p_{i2})$. The axes direction of P joint denoted as $s_{ip}=(x_i,y_i,z_i)$.

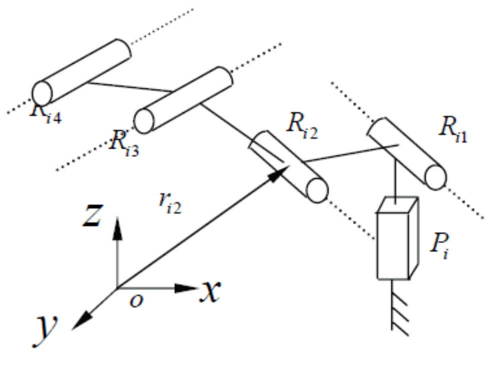

Figure 1 The structure of mechanism

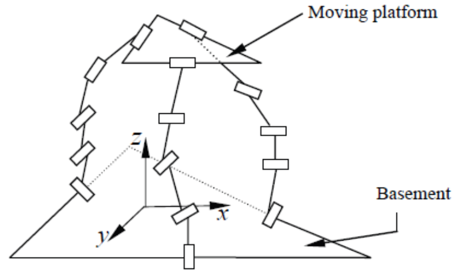

Figure 2 Kinematic model of the ith limb

The linear vector from the original point O to a point of the axes denoted as $r_{im}=(x_{im},y_{im},z_{im})(m=1\sim 3)$. The screw system of the i^{th} can be derived as following:

$$\$_{pi}=(0,s_{pi}) \quad \$_{i,r1}=(s_{i1},r_{i,r1}\times s_{i1})$$
$$\$_{i,r2}=(s_{i1},r_{i,r2}\times s_{i1})$$
$$\$_{i,r3}=(s_{i2},r_{i,r3}\times s_{i2})$$
$$\$_{i,r4}=(s_{i2},r_{i,r4}\times s_{i2})$$

The reciprocal screw of Eq.(4) is

$$\hat{\$}_i=(0,0,0,\frac{n_{i2}p_{i1}-n_{i1}p_{i2}}{m_{i2}n_{i1}-m_{i1}n_{i2}},\frac{m_{i1}p_{i2}-m_{i2}p_{i1}}{m_{i2}n_{i1}-m_{i1}n_{i2}},1) \quad (5)$$

Considering three reciprocal screws of limbs, we can derived a matrix with 3 by 3, as

$$Matrix=[N_i,M_i,1](i=1\sim 3) \quad (6)$$

where

$$N_i=\frac{n_{i2}p_{i1}-n_{i1}p_{i2}}{m_{i2}n_{i1}-m_{i1}n_{i2}} \quad M_i=\frac{m_{i1}p_{i2}-m_{i2}p_{i1}}{m_{i2}n_{i1}-m_{i1}n_{i2}}$$

Three cases can be taken into account.

Case one: when $m_{i2}n_{i1}-m_{i1}n_{i2}=0$, that is all the axes lie in planes paralleled with each other. The Eq.(5) changes to another form, denotes as

$$\hat{\$}_i=(0,0,0,-\frac{n_{i2}}{m_{i2}},1,0) \quad (7)$$

Obviously, three is structure singularity in this case as all of the three reciprocal screws coplanar and the max linear independent of these pure couple is two, as shown in Fig. 3.

Case two. When the range of sub-matrix equals to zero, there is another singularity form. That is to say two of the three reciprocal screws are linear dependent (intersect or parallel each other), as shown in Fig. 4.

Case three. When the range of matrix equals to zero, that is

$$Det(Matrix)=|N_i,M_i,1|=0(i=1\sim 3)$$

It can be denoted as following.

$$M_2N_1 + M_3N_2 + M_1N_3 - M_3N_1 - M_1N_2 - M_2N_3 = 0 \quad (8)$$

Any configuration of the axes of revolute joints will be structure singular when satisfy the Eq.(8), as shown in Fig. 5.

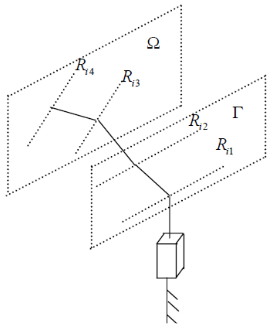

Figure 3 Axis of each revolute joint lie in the xy planar

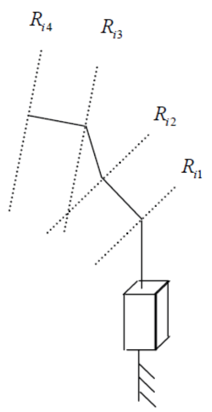

Figure 4 The axis of two groups revolute joints in one limb intersected each other

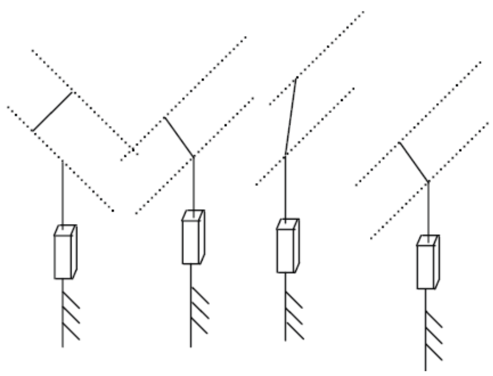

Figure 5 The axis of revolute joint in one limb intersect or parallel with the axis of revolute joint in another limb

B Actuated singularity

The architecture of the most general spherical three-degree-of-freedom parallel manipulator with prismatic actuators was presented by Gosselin and Lavoie [10], as a robotic wrist or joystick. Actuated singularity includes its configuration singularity and kinematic singularity. Here we are interested only in the configuration singularity of three-degree-of-freedom P-4R parallel manipulator with prismatic actuators which are mounted on the base.

Assuming the angle which axes of prismatic actuated joints interests with the planar **xy** is θ_i, and the angle interests with x axis is α_i. The matrix composed by axes of actuated joints can be denoted as

$$A = \begin{bmatrix} c\theta_1 c\alpha_1 & c\theta_1 s\alpha_1 & s\theta_1 \\ c\theta_2 c\alpha_2 & c\theta_2 s\alpha_2 & s\theta_2 \\ c\theta_3 c\alpha_3 & c\theta_3 s\alpha_3 & s\theta_3 \end{bmatrix} \quad (9)$$

where $c[\bullet] = Cos[\bullet];\ s[\bullet] = Sin[\bullet]$.

The actuated singularity occurs when the rank of matrix A equals to zero.

$$Det(A) = c\alpha_1 c\theta_1 c\theta_2 s\alpha_2 s\theta_3 + c\alpha_2 c\theta_2 c\theta_3 s\alpha_3 s\theta_1 + c\alpha_3 c\theta_1 c\theta_3 s\alpha_1 s\theta_2 - c\alpha_3 c\theta_2 c\theta_3 s\alpha_2 s\theta_1 - c\alpha_1 c\theta_1 c\theta_3 s\alpha_3 s\theta_2 - c\alpha_2 c\theta_1 c\theta_2 s\alpha_1 s\theta_3 \quad (10)$$

Case one. α_i of each actuated joint is equal, Eq.(10) is equal to zero, that is, there is actuated singularity. As showed in Fig. 6.

Case two. If $\theta_i = 0$, it is said that all of the axes of the actuated joints lie in the planar **xy**. The third arrange of matrix A equals to zero, and rank of matrix A is also equal to zero. This is another actuated singularity. As showed in Fig. 7.

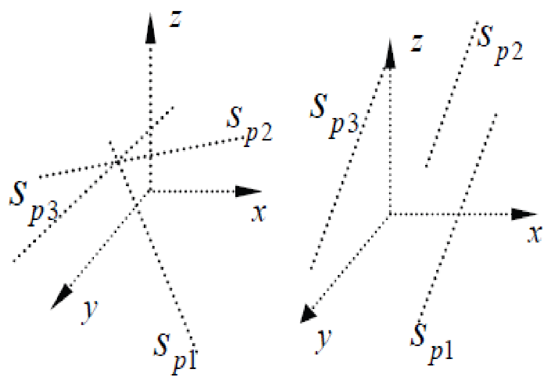

Figure 6 Axis of actuated joints intersect or parallel each other in space

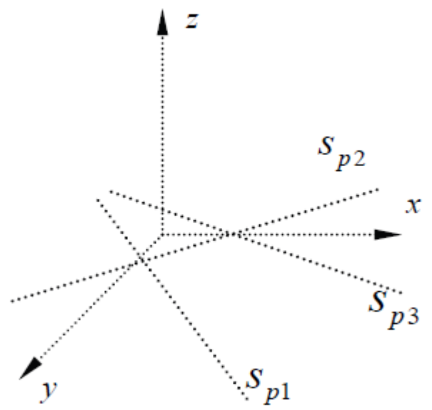

Figure 7 Three axis of the actuated joints lie in one planar(xy)

III. SUMMARY OF SINGULARITY

Analysis as previous, there are two kinds of singularity in this mechanism: structure singularity and actuated singularity. Structure singularity occurs when the displacement of each limb lies in different certainly position. This position includes: axes of each revolute joint lies in the xy planar; the axis of two groups revolute joints in one limb interested each other; the axis of revolute joint in one limb interests or is parallel with the axis of revolute joint in another limb. Actuated singularity occurs when the actuated prismatic joint configures in different displacement, including: two or three axes of the actuated joints are interested or parallel with each other in space; three axes of the actuated joints lie in one planar**(xy)**. As showed in figures.

IV. CONCLUSION

Using the screw theory, reciprocal screw, and linear geometric theory, a set of explicit algorithms for both structure singularities and actuated joints configuration singularities analysis have been developed for a new family of 3-DoF P-4R parallel manipulators. Since the singularity analysis algorithms are based on the translational movement and pure couple restraint of joints, so we only need to deal with 3×3 matrixs. Three types of structure singularities and two types of actuated configuration singularities have been identified for a new family of 3-DoF P-4R parallel manipulators. Figures given in section 3 described the displacement of different singularities. These geometric conditions can be used as guildlines for the design of parallel mechanisms that can be avoid architecture singularities. The effectiveness method can be used to analyze the same special parallel manipulators. Based on these analysis theory, we can do advance research and analyze the kinematical mode of these mechanisms.

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